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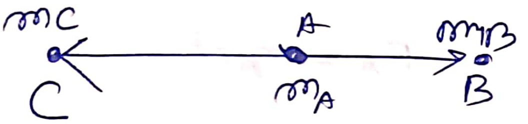
Decay of unstable particle in the Context of four vector: (The phenomenon is observed w.r. to the rest frame of a parent particle.

Let a particle of mass m_A, decays to two particle of rest mass m_B and m_C resp.



Let the relativistic energies of the particles are E_A, E_B and E_C resp.

In the context of four vector Let the momentum of the three particle are

$$P_A^{\mu} = \left(\frac{E_A}{c}, 0, 0, 0\right)$$

$$P_B^{\mu} = \left(\frac{E_B}{c}, P_B^1, P_B^2, P_B^3\right)$$

& $$P_C^{\mu} = \left(\frac{E_C}{c}, P_C^1, P_C^2, P_C^3\right)$$

Applying the conservation ~~fo~~ of, ~~E~~four momentum vector

$$P_A^{\mu} = P_B^{\mu} + P_C^{\mu}$$

$$\Rightarrow P_C^{\mu} = P_A^{\mu} - P_B^{\mu}$$

Taking the inner product ~~from~~ in the both side

$$(P_C^{\mu})^2 = (P_A^{\mu} - B_B^{\mu})^2$$

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$$\Rightarrow -m_c^2c^2 = (P_A^\mu)^2 + (P_B^\mu)^2 - 2P_A^\mu P_B^\mu$$

$$\Rightarrow -m_c^2c^2 = -m_A^2c^2 - m_B^2c^2 - 2P_A^\mu P_B^\mu$$

$$\Rightarrow -m_c^2c^2 = -m_A^2c^2 - m_B^2c^2 + 2\frac{E_A E_B}{c^2}$$

$$\Rightarrow -m_c^2c^2 = -m_A^2c^2 - m_B^2c^2 + 2m_A E_B$$

$$\therefore E_B = \frac{(m_A^2 + m_B^2 - m_c^2)c^2}{2m_A}$$

&

$$E_C = \frac{(m_A^2 + m_c^2 - m_B^2)c^2}{2m_A}$$

$(P^\mu)^2 = -m_0^2c^2$ [momentum square]

Now $P_A^\mu P_B^\mu = -\frac{E_A}{c}\frac{E_B}{c}$

$E_A = m_A c^2$

$$\therefore P_A^\mu P_B^\mu = -\frac{m_A c^2 E_B}{c^2} = -m_A E_B$$

Q what is unstable particle?

Ans ① unstable Particles are those particle which can decay into two or more particles. As for example a μ meson is an unstable particle that can decay spontaneously into an electron and two neutrinos.

Q. Get the relation betn the rest masses of the decaying nucleus and the sum of the rest masses of the daughter nucleus.

Ans.

Let M is the rest mass of the decaying nucleus and m_1 and m_2 be the rest masses of the daughter nucleus.

we have the kinetic energy of the daughter nucleus = Change in rest mass energy

$$\therefore \frac{1}{2}m_1c^2 + \frac{1}{2}m_2c^2 = \{M - (m_1+m_2)\}c^2$$

$$\therefore \frac{1}{2}m_1c^2 + \frac{1}{2}m_2c^2 + (m_1+m_2)c^2 = Mc^2$$

$$\Rightarrow \frac{3}{2}(m_1+m_2)c^2 = Mc^2$$

$$\Rightarrow M = \frac{3}{2}(m_1+m_2)$$

• This is the relation between sum of the rest masses of the daughter nucleus and the mass of the decaying nucleus. ②

② It is also clear that the mass of the decaying nucleus is greater than the sum of the masses of the daughter nucleus.

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Q A pi meson at rest decays in to a muon and neutrino. Find the momentum of the muon in terms of the rest masses of the π meson and μ muon.

Before the decay $E_i = m_\pi c^2$

$p_i = 0$

After decay $E_f = E_\mu + E_\nu$

$p_f = p_\mu + p_\nu$

As $p_i = p_f$ $\therefore p_\mu = -p_\nu$

$\Rightarrow p_\mu + p_\nu = 0$ $\longrightarrow$ (I)

& $E_\mu + E_\nu = m_\pi c^2$ $\longrightarrow$ (II)

} Following the conservation of momentum & energy.

As the rest mass of the neutrino is zero.

$\therefore E_\nu = |p_\nu| c \Rightarrow p_\nu = E_\nu / c = -p_\mu$

NO Need

$\left[\therefore |p_\mu| = \dfrac{\sqrt{E_\mu^2 - m_\mu^2 c^4}}{c} \right.$ $\longrightarrow$ (III) using the relation $E^2 = p^2c^2 + m_0^2 c^4$

$\therefore E_\nu = |p_\mu| c$

$= \sqrt{E_\mu^2 - m_\mu^2 c^4}$ using this in (II)

$\sqrt{E_\mu^2 - m_\mu^2 c^4} = m_\pi c^2 - E_\mu \longrightarrow$ using in (III)

$\left. \therefore p_\mu = m_\pi c^2. \right]$

$\therefore \; P_\mu = -E_\gamma / c \longrightarrow$ (III)

From equ^n (II)

$$E_\gamma = m_\pi c^2 - E_\mu \longrightarrow \text{(IV)}$$

And $$E_\mu = \frac{(m_\pi^2 + m_\mu^2)c^2}{2m_\pi} \longrightarrow \text{(V)}$$

As $m_\gamma = 0$

using (IV) and V in (III)

$$P_\mu = \{E_\mu - m_\pi c^2\}/c$$

$$= \left\{\frac{(m_\pi^2 + m_\mu^2)c^2}{2m_\pi} - m_\pi c^2\right\}/c$$

$$\boxed{P_\mu = \frac{(m_\mu^2 - m_\pi^2)c}{2m_\pi}}$$

Q what is the kinetic energy of a particle moving with speed of light.

Ans we have the kinetic energy of a particle is

KE = Total energy − rest mass energy

$$= mc^2 - m_o c^2$$

$$= \frac{m_o c^2}{\sqrt{1 - \frac{v^2}{c^2}}} - m_o c^2 = m_o c^2 \left\{\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1\right\}$$

when the particle moves with speed of light

$v = c \; \therefore \quad KE = \infty$

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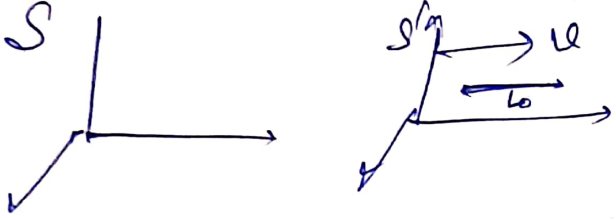
Q. If total charge is invariant, how can it be that a neutral wire in one frame appears to be charged in other.

Ans

(*) Let there are two frames of reference S and S' where S' is moving with a const. translational velocity v with respect to S



Let in the S' frame there is a wire of length L_0, which is rest and neutral with respt to S.

As the frame S' is moving so the length of the wire will seem to be contracted to $L_0\sqrt{1-\frac{v^2}{c^2}}$

Due to the change in the length of the wire, the distribution of charge will also change and the wire behaves as a charged one w.r. to S.

Covariance:

The term covariance implies a formalism in which the laws of physics maintain the same form under specific transformation.

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Q. Write the Maxwell equation in Four-D form.

Ans. We have the set of Maxwell equation is,

$$\left.\begin{aligned} \nabla \cdot E &= \rho/\epsilon_0 \\ \nabla \cdot B &= 0 \\ \nabla \times E &= -\frac{\partial B}{\partial t} \\ \nabla \times B &= \mu_0 J + \epsilon_0 \mu_0 \frac{\partial E}{\partial t} \end{aligned}\right\} \quad (1)$$

E is the electric field and B is the magnetic field and the E and B can be expressed as

$$\left.\begin{aligned} E &= -\nabla V - \frac{\partial A}{\partial t} \\ B &= \nabla \times A \end{aligned}\right\} \quad (2)$$

V and A together constitute a four vector

$$A^{\mu} = \left(\frac{V}{c}, A_x, A_y, A_z\right) \longrightarrow (3)$$

In terms of these four vector the field equation in tensor form can be written as

$$F^{\mu\nu} = \frac{\partial A^{\nu}}{\partial x_{\mu}} - \frac{\partial A^{\mu}}{\partial x_{\nu}} \longrightarrow (4)$$

The differentiation is done in terms of x_μ and x_ν covariant vector, the zeroth component of covariant ~~covent~~ covariant and contravariant vector changes their signs in case of zeroth component $x_0 = -x^0$

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we have to varify that equation (4) is equivalent to equation (2), let $\mu = 0$ & $\nu = 1$

$$\therefore F^{01} = \frac{\partial A^1}{\partial x_0} - \frac{\partial A^0}{\partial x_1} = -\frac{\partial A_x}{\partial (ct)} - \frac{1}{c}\frac{\partial V}{\partial x}$$

$$= -\frac{1}{c}\left(\frac{\partial A}{\partial t} + \nabla V\right)_x = \frac{E_x}{c}$$

Now $\mu = 1, \nu = 2$

$$F^{12} = \frac{\partial A^2}{\partial x_1} - \frac{\partial A^1}{\partial x_2} = \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} = (\nabla \times A)_z = B_z$$

$$F^{23} = \frac{\partial A^3}{\partial x_2} - \frac{\partial A^2}{\partial x_3} = \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} = (\nabla \times A)_x = B_x$$

$$F^{13} = \frac{\partial A^1}{\partial x_3} - \frac{\partial A^3}{\partial x_1} = \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} = (\nabla \times A)_x = B_y$$

(5)

Equation (5) is the second equation of equation (2).

we have the combined transformation relation for the em field in tensor form can be written as

$$F^{\mu\nu} = \begin{bmatrix} 0 & E_x/c & E_y/c & E_z/c \\ -E_x/c & 0 & B_z & B_y \\ -E_y/c & -B_z & 0 & B_x \\ -E_z/c & B_y & -B_y & 0 \end{bmatrix} \rightarrow (6)$$

As for the inhomogeneous equation

$$\frac{\partial F^{\mu\nu}}{\partial x^\nu} = \mu_0 J^\mu \longrightarrow (7)$$

That becomes $\frac{\partial}{\partial x_\mu}\left(\frac{\partial A^\nu}{\partial x^\nu}\right) - \frac{\partial}{\partial x_\nu}\left(\frac{\partial A^\mu}{\partial x^\nu}\right) = \mu_0 J^\mu \longrightarrow (8)$

This is an intractable equation. ~~as it stand~~.

* [[Not to be included in the ans:- while studying the scalar and vector potential you have learned that the scalar and vector potentials are not uniquely ~~defined by the~~ determined by the fields.]]

from equation ~~(4)~~ (4) it is clear that one could add to A^μ the gradient of any scalar function λ.

$$\therefore A^\mu \longrightarrow A^{\mu'} = A^\mu + \frac{\partial \lambda}{\partial x_\mu} \longrightarrow (9)$$

This change does not change $F^{\mu\nu}$.

In particular the relation betn scalar and vector potential, known as Lorentz gauge condition

$$\nabla \cdot A = -\frac{1}{c^2}\frac{\partial V}{\partial t},$$

in relativistic relation becomes $\frac{\partial A^\mu}{\partial x^\nu} = 0$

The eqn (8) reduces to

$$\Box^2 A^\mu = -\mu_0 J^\mu \longrightarrow (10)$$

$$\Box^2 \equiv \frac{\partial}{\partial x_\nu}\frac{\partial}{\partial x^\nu} = \nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} \longrightarrow (11)$$

Equation (10) is the four vector formalism of Maxwell equation

Rough

$$\nabla \cdot A + \frac{1}{c^2}\frac{\partial V}{\partial t} = 0$$

$$\frac{\partial V}{\partial (ct)} + \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} = 0$$

$$\frac{\partial A^0}{\partial x^0} + \frac{\partial A^1}{\partial x^1} + \frac{\partial A^2}{\partial x^2} + \frac{\partial A^3}{\partial x^3} = 0$$

$$= \frac{\partial A^\mu}{\partial x^\nu} = 0$$

where $A^0 = V/c$

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